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The Application of Functions of Several Variables Analysis in an Optimal Replenishment Policy for Deteriorating Items

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Abstract

Very often must be taken into account the gradual deterioration of goods in inventory management. This fact is not taken into account in multiple management systems. Traditional inventory models at the same time assume that a retailer pays for the goods the moment they are received. Nowadays, however, it is becoming a common practice that a supplier offers a retailer the option to pay for the goods with a certain delay. If the retailer is not able to meet his obligations within the deadline, he is charges an interest. In this study we introduce a newly constructed suitable model which enables a retailer to set an optimal price of deteriorating goods under permissible delay in payments, and to determine the maximum repayment term. We considered a deterministic inventory model with time-dependent demand, holding costs variable in time where deterioration is directly proportional to the time. The model is based on the assumption of time-dependent demand and has been developed for deteriorating goods. The paper further analyses a situation in which the retailer sell all the goods in time, and a situation in which the deadline was not met. Further assumption is that the inventory is depleted only by demand. The scientific aim is to verify if such an optimizing problem can be solved. Theoretical results are illustrated with numerical example for the model. Results show that the developer model is capable of solving the theoretical problem illustrated by an example. It helps to the retailer to set the selling price and the replenishment interval in order to maximize profit. The authors of the paper used methods of analysis and synthesis, and the method of mathematical analysis (differential calculus of multivariable functions, solution of ordinary differential equations, Taylor series). The model suggested in the paper can be expanded in the future. One option is generalization of the model, allowing for the lack of goods, bulk discounts, time value of money, inflation etc.

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Keywords: inventory management, EOQ model, deteriorating goods, multivariable functions;

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1. Introduction

In today's world, more and more emphasis is put on increasing the productivity of work, effectiveness of management processes and all the other activities taking place within an enterprise. In spite of their generally positive role in a modern enterprise, inventories are usually thought of as a reserve in the managers´ work, and ways of reducing inventory level are looked for. Due to the fact that the level of inventories is easily measurable and that modern computer technologies enable us to monitor the inventory level in entire supply chains, inventory management is in the centre of attention of people specializing in the application of mathematical methods to enterprise management.

Traditional inventory models assume that the trader pays for the goods he has purchased at the moment when it is moved into stock. However, it is currently a common practice that the supplier offers the trader the possibility of paying for the goods with a certain delay. Until the end of this period, the trader is able to sell the goods and withhold the money on his account and thus gain interest on it. If he is unable to pay for the goods within the contractual period, he is charged an interest. In other words, the supplier grants the trader an interest-free loan for the contractual period.

This article is aimed at constructing a mathematical model allowing the trader to determine (based on the knowledge of certain parameters) the optimal selling price per item and the maximum time interval for which the goods can be sold with profit. The model is based on the assumption of a time-dependent demand and developed for deteriorating items with a specified deterioration rate. Also, it is assumed that inventory is drawn on the basis of demand only. From the scientific point of view, our goal is to verify whether such an optimization problem is solvable. To achieve this goal we have used the methods of mathematical analysis (differential calculus of several variables, solution of ordinary differential equations).

The new model created in this article is illustrated by a concrete problem and the solution is presented in graphical form.

2. Literature research

The basic EOQ model is based on the implicit assumption that retailer must pay for the items as soon as he receives them from a supplier. However, a common practice in industries is to provide a specific delay period for the payments after the items are delivered. In this regard, a number of research papers appeared which deal with the EOQ problem under fixed credit period. Whitin (1955) was the first researcher to extend the basic EOQ model by considering the selling price in addition to the order quantity as the decision variables.

Deterioration is a fact of life in inventory items, such as volatile liquids, agricultural products, radioactive substances, films, drugs, blood, fashion goods, electronic components and high-tech products. These items are subject to depletion by phenomena other than demand, i.e. through spoilage, shrinkage, decay and obsolescence. Ghare and Schrader (1963) extended the classical EOQ model by considering the exponentially decaying inventory when the demand is constant. Covert and Philip (1973) developed an economic lot-size model for situation in which the deterioration follows a Weibull distribution, under the assumptions of a constant demand rate with no shortages allowed. Hariga (1996), Teng, Yang, and Ouyang (2003) and Wu (2002) also extended EOQ-based deteriorating inventory models by considering a time-varying demand function, with or without shortages and some with partial backlogging. Manna and Chaudhuri (2001) discussed an EOQ model with deteriorating items in which the production rate is proportional to the time dependent demand rate.

Goyal (1985) developed an EOQ model under conditions of permissible delay in payments. He ignored the difference between the selling price and the purchase cost, and concluded that the economic replenishment interval and order quantity generally increases marginally under the permissible delay in payments.

Although Dave (1985) corrected Goyal's model by assuming the fact that the selling price is necessarily higher than its purchase price, his viewpoint did not draw much attention to the recent researchers. Aggarwal and Jaggi (1995) then extended Goyal's model for deteriorating items. Jamal et al. (1997) further generalized the model to allow for shortages and deterioration. Hwang and Shinn (1997) developed the optimal pricing and lot sizing for the retailer under the condition of permissible delay in payments. Liao et al. (2000) developed an inventory model for stock-depend demand rate when a delay in payment is permissible. Recently, Chang and Dye (2001) extended the model by Jamal et al. to allow for not only a varying deterioration rate of time but also the backlogging rate to be inversely proportional to the waiting time. All above models (except Dave, 1985) ignored the difference between unit price and unit cost, and obtained the same conclusion as in Goyal (1985). Incontrast, Jamal et al. (2000) and Sharker et al. (2000) amended Goyal's model by considering the difference between unit price and unit cost, and concluded from computational results that the retailer should settle his account relatively sooner as the unit selling price increases relative to the unit cost. Recently, Teng (2002) provided an alternative conclusion from Goyal (1985), and mathematically proved that it makes economic sense for a well-established buyer to order less quantity and take the benefits of the permissible delay more frequently. Chang et al. (2003) then extended Teng's model, and established an EOQ model for deteriorating items in which the supplier provides a permissible delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity.

Lately, some researches discussed the impact of delay payment strategy on the inventory models. Abad and Jaggi (2003) provided a seller – buyer inventory model under trade credit and followed a lot for-lot shipment policy. Jaber and Osman (2006) proposed a supplier – retailer supply chain model in which the permissible delay in payments is considered as a decision variable. Yang and Wee (2006) developed a single vendor and single buyer collaborative inventory system for deteriorating items with permissible delay in payment.

Sana and Chaudhuri (2008) developed a model the retailer‘s profit-maximizing strategy when confronted with supplier‘s trade offer of credit and price-discount on the purchase of merchandise. Hong, Huo, and Li Zhao Xia (2008), in their paper consider an EOQ model for deteriorating items when delay in payment is permitted by supplier to retailer. Moreover, in this model, the supplier offers cash discount on the purchasing cost to the retailer.

Inventory management by Nenes, Panagiotidou and Tagaras (2010) has been recognized as one of the most important functions of industrial and commercial enterprises, which often has a great impact on their overall performance.

3. Assumptions of the Model

In this chapter, a convenient model is proposed, which allows the trader to determine the optimal price of the goods in a situation where the supplier grants the trader a permissible delay in payment, and the maximum possible delay in payment. The model is constructed with the assumption of a time-dependent demand and developed for deteriorating goods. Also, it is assumed that inventory is only drawn on the basis of demand.

Our additional assumptions are:

- Demand for the goods is a decreasing function of the price and time variables
- Shortages are not allowed
- Replenishments are instantaneous
- Planning horizon is infinite
- Deterioration rate is constant

In the sequel, the following variables are used:

Nomenclature

H	unit holding cost per year (excluding interest charges)
c	purchasing price per unit, $c > 0$
p	selling price per unit, $c < p$
θ	deterioration rate; $0 < \theta < 1$
I_d	received interest per currency unit per year
I_c	annual interest on late payment per currency unit in stock
m	period of permissible delay in settling the account – period of supplier (commercial) credit
s	ordering cost per order
Q	order quantity
I(t)	on-hand inventory at time t ($0 \le t \le T$)

T	replenishment interval $T > 0$
D	annual demand, dependent on time and price per unit, $D(p,t) = \alpha p^{-\beta} t$, where we assume that $\alpha > 0$ and $\beta > 0$, α is a scaling factor, and β is a price-elasticity coefficient. (Ho et al. (2008)). For simplicity, we use the notation $a = \alpha p^{-\beta}$ in the following text.
Z(T,p)	total annual profit.
A	radius of
B	position of
C	further nomenclature continues down the page inside the text box

The total annual profit is made up of income from sales, ordering costs, purchasing costs, costs caused by deterioration of items, storage costs (excluding the interest charges), interest paid on goods which has not been sold within the permissible delay in payment (for $T > m$ only), received interest on income from sales before the lapse of the permissible delay in payment.

4. Initial Assumptions

We can assume that the inventory level $I(t)$ gradually decreases with time to satisfy the demand. Then the change of inventory level as a function of time is defined by the following differential equation:

$$\frac{dI(t)}{dt} + \theta I(t) = -D(p,t),\ 0 \le t \le T \tag{1}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -at, 0 \le t \le T, a = \alpha p^{-\beta} \tag{2}$$

The initial condition is defined by $I(T) = 0$.

Thus the trader sells $\frac{aT^2}{2}$ items per one business cycle and $\frac{aT^2}{2}$ items per year. He purchases $\frac{a(-e^{\theta T}+\theta T e^{\theta T}+1)}{\theta^2}$ items per one cycle, which represents $\frac{a(-e^{\theta T}+\theta T e^{\theta T}+1)}{T\theta^2}$ items per year.

He must settle his supplier credit in full, and therefore pays a total of $\frac{ca(-e^{\theta T}+\theta T e^{\theta T}+1)}{\theta^2}$, which is $\frac{ca(-e^{\theta T}+\theta T e^{\theta T}+1)}{T\theta^2}$ per year.

As there are many different ways of determining the cost and yield interest, the Goyal approach, as published in (Goyal, 1985), is used in the following text.

4.1. Analysis of the situation where the items are sold on time

If $T \le m$, the trader has no additional costs and he can even gain interest in the amount of $pI_d(\frac{aT^2(T-m-1)}{2})$. This implies that he will gain interest in the amount of $pI_d(\frac{aT(T-m-1)}{2})$ per year.

The total annual profit can then be expressed by the formula

$$Z1(T,p) = \frac{paT}{2} - \frac{s}{T} - \frac{ca(-e^{\theta T}+\theta T e^{\theta T}+1)}{T\theta^2} - \frac{Ha(-T^2\theta^2+2e^{\theta T}T\theta-2e^{\theta T}+2)}{2\theta^3} + pI_d\left(\frac{-aT(T-m-1)}{2}\right) \tag{3}$$

Using the Taylor expansion (the first three terms of the series) we can rewrite the equation in a form which represents a great simplification for subsequent numerical computations

$$Z1(T,p) = \frac{paT}{2} - \frac{s}{T} - \frac{ca\left(-\left(1+\theta T+\frac{\theta^2T^2}{2}\right)+\theta T\left(1+\theta T+\frac{\theta^2T^2}{2}\right)+1\right)}{T\theta^2} + pI_d\left(\frac{-aT(T-m-1)}{2}\right)$$

It is obvious from (3) that the function $Z1$ is a continuous function of the two variables, in particular for any $T > 0$ and $p > 0$.

In modeling our economic process it is meaningful to limit ourselves to the values of the variables T and p contained in the rectangle $O = \{[T,p] \mid T \epsilon [1, Tmax], p \epsilon [c, pmax]\}$, which represents a so-called compact set in the plane. Obviously, the function $Z1$ is also continuous in this rectangle. Thus it follows from the so-called Weierstrass theorem (see the Differential calculus of several variables – e.g. Došlá and Kuben (2012)) that the function $Z1$ attains its maximum and minimum values in the rectangle O. Moreover, these values may be attained either at the points of local extremes (in the interior of the rectangle) or on its boundary.

Since

$$\frac{\partial^2 \mathrm{Z1(T,p)}}{\partial \mathrm{T}^2} = -\frac{\mathrm{ca\theta T^3 + HaT^3 + I_d apT^3 + 2s}}{\mathrm{T^3}} < 0 \tag{4}$$

the function $Z1$ is concave in the rectangle O. Therefore, if it attains a local extreme in this rectangle, the local extreme is the maximum value of the function.

4.2. Analysis of the situation where the items are sold on time

If $T > m$, the trader must take a credit at the moment m , on which credit he will pay an interest in the amount of $\frac{I_c pa(m^2\theta^2 + 2T\theta e^{\theta(T-m)} - 2m\theta - T^2\theta^2 - 2e^{\theta(T-m)} + 2)}{2\theta^3}$.

This implies that he will pay interest charges in the amount of $\frac{I_c pa(m^2\theta^2 + 2T\theta e^{\theta(T-m)} - 2m\theta - T^2\theta^2 - 2e^{\theta(T-m)} + 2)}{2T\theta^3}$ per year (Goyal, 1985).

Nevertheless, he may gain interest on a deposit from the sales received before the moment m, namely interest in the amount of $\frac{\mathrm{am^2 I_d}}{2}$, which represents a total of $\frac{\mathrm{am^2 I_d}}{2\mathrm{T}}$ per year.

In this case, the total annual profit may be expressed by the formula

$$\mathrm{Z2(T,p)} = \frac{\mathrm{paT}}{2} - \frac{\mathrm{s}}{\mathrm{T}} - \frac{\mathrm{ca(-e^{\theta T} + \theta Te^{\theta T} + 1)}}{\mathrm{T\theta^2}} - \frac{\mathrm{Ha(-T^2\theta^2 + 2e^{\theta T}T\theta - 2e^{\theta T} + 2)}}{2\theta^3}$$
$$- \frac{\mathrm{I_c pa(m^2\theta^2 + 2T\theta e^{\theta(T-m)} - 2m\theta - T^2\theta^2 - 2e^{\theta(T-m)} + 2)}}{2\mathrm{T}\theta^3} + \frac{\mathrm{am^2 I_d}}{2\mathrm{T}} \tag{5}$$

Using the Taylor expansion again (the first three terms of the series) we rewrite the equation in a form which means a considerable simplification for the subsequent numerical calculations.

Further, we observe that

$$\frac{\partial^2 \mathrm{Z1(T,p)}}{\partial^2 \mathrm{T}} = -\frac{\mathrm{6s + 3a\theta T^3 + 3HaT^3 + 3I_c apT^3 - 2I_d apm^2}}{3} < 0, \tag{6}$$

and therefore the function $Z2(T,p)$, under the above-specified conditions, is a concave function and its local extreme is the maximum of the function $Z2$.

5. Illustrative example

Let's assume the following values of a fictitious business:

$H = 3$ (units per year), $I_c = 0.2$ (currency unit per year), $I_d = 0.002$ (currency unit per year), $c = 0.2$ (currency unit), $s = 5$ (currency unit per item), $\alpha = 10\ 000$, $\beta = 1.6$, $m = 15/365$ (year), deterioration rate $\theta = 0.07$.

Our problem is to determine the optimal price per item and the maximum time of sale allowing the trader to generate a profit. In accordance with the above, we assume that $p > c$ and that the items may be sold on the first day at the earliest, i.e. $T > 0$.

5.1. Software used in solving the problem

All computations have been made by the Maple system which is a mathematical software popular for its ability to perform computations in symbolic form. The software is similar to the Mathematica and Maxima programs, which however offer much fewer functions. The advantage of Maple over these systems is that not only it performs analytical computations with formulas, but handles equally well numerical computations and graphical representations, providing a wide range of possibilities for the use of quantitative methods in practice, application problems, scientific computations for a number of fields etc.

5.2. Situation where the goods is sold on time

If the trader assumes that the items will be sold under specified conditions and on time, he/she maximizes his/her income at $T = 15/365$ and $p = 0.862$. It follows from our results that the maximization of profit will be achieved at $T = m$, which means that the goods should be sold at the end of the period of permissible delay. The graph in Fig. 1 shows the area representing all possible situations which may occur under the specified conditions, assuming the trader does not incur a loss.

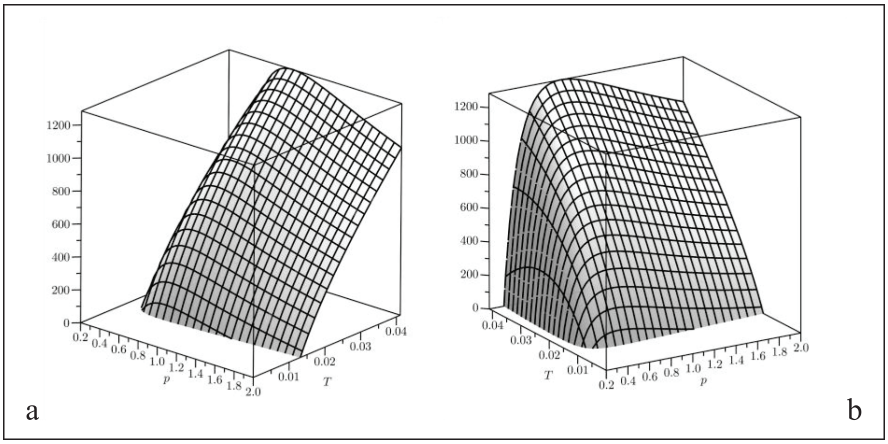

Fig. 1a-b. Trader's possible income in the case $T \leq m$ (Source: Author).

The graph in Fig. 2 allows to find out what profit the trader can make with different selling prices, provided the items are sold exactly at the end of the permissible delay in payment. The graph also allows to estimate the optimal price.

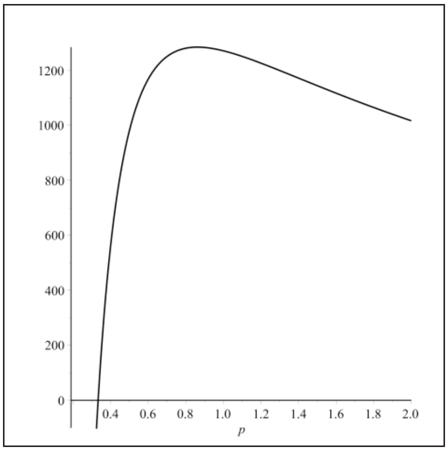

Fig. 2. Trader's possible income for T = m (Source: Author).

5.3. Situation where the goods will be sold after the due date of the obligation

Let us now consider a situation where the trader knows that he will not be able to meet the due date. In this case, the price p is determined by maximizing the function $Z2$, because the trader must pay late charges beginning from time $T > m$. The trader is able to make a maximum profit at the price $p = 11.615$ and the credit is settled at time $T = 1.088$, i. e. after a period of 1 year and 32 days.

5.4. Situation where the goods is unexpectedly sold after due date of the obligation

Let us now consider a situation where the trader at the start expects to sell the goods „on time", but in fact fails to meet the due date. In that case, the price p is determined in the same way as in the first case, i. e. $p = 0.862$, but the trader must also pay late charges from time $T = m$. This situation is graphically represented in Fig. 3, which shows all possible situations which may arise under the specified conditions, with $p < 0.862$. In this case, however, the trader runs the risk that his costs will exceed his incomes, namely at time $T = 0.211$, which means that the trader may incur a loss at price $p = 0.862$, unless he sells the goods within 77 days. The situation in the case of a profit is illustrated in Fig. 4.

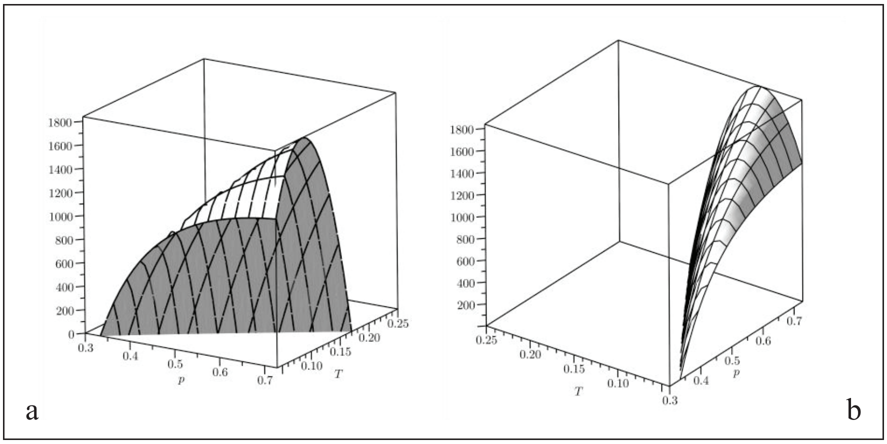

Fig. 3a-b. Trader's possible income for T > m, p ≤ 0.862 (Source: Author).

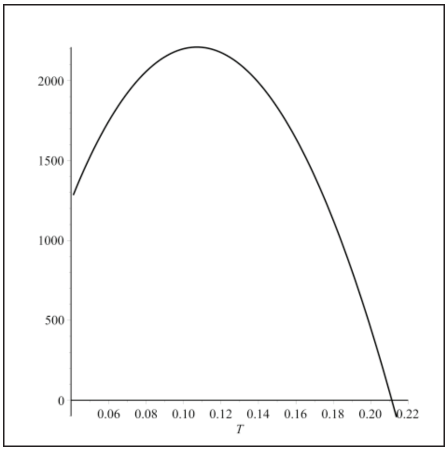

Fig. 4. Trader's possible income for T > m, p = 0.862 (Source: Author)

5.5. Effect of change of certain parameters on model behaviour

The exactness of our model and the accessibility of suitable software enable us to easily determine what effect a change of external factors would have on the model. To illustrate such changes we have chosen the change of interest charged I_c and the change of purchasing price c.

5.6. Effect of a change in the parameter I_c on the time of sale

An increase of the interest charged I_c means that if the trader fails to meet the due date, he/she must sell the goods considerably faster, otherwise he/she would have to pay a higher interest and thus lose a part of his/her profit. If we consider the value $I_c = 0.3$ (while keeping all the other parameters at the original level), then the critical time of sale is $T = 0.751$ at $p = 8.225$. The graphical representation can be seen in Fig. 5. In contrast, a decrease of the value I_c to 0.15 causes an increase of the time for which the product can be sold without the trader risking a loss, to the absurd value $T = 1.423$ at $p = 14.987$.

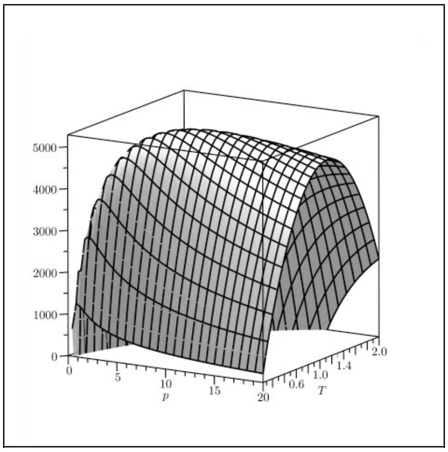

Fig. 5. Trader's possible income for Ic = 0.3 (Source: Author)

5.7. Effect of change of the parameter c on the time of sale

An increase of the purchasing price c to the value $c = 0.15$ causes the optimal selling price, under the assumption of a timely payment, to decrease to the value $p = 0.728$, making it possible to postpone the sale of goods until after the due date for the time $T = 0.137$, i. e. by 50 days. In contrast, increasing the price c to the value 0.3 allows (with p at 12.347) the trader to sell the goods with a profit for the time $T = 2.237$. Again, these changes can be observed while keeping all the other parameters at the original level.

6. Conclusion

The article presents a model for calculating the optimal price and the maximum possible due date in the case of a credit on goods, where the supplier grants the trader a permissible delay in payments. The model has been developed for deteriorating items. The article analyses situations where the trader sells all the goods on time as well as situations where he is unable to meet the delay for selling the goods and has to pay interest charges on his credit.

Theoretical results are clearly illustrated by an example of a fictitious business and concrete results are presented in graphical form.

The model proposed in the article offers the possibility of a future extension. One of the possibilities of extending the model consists in a generalization allowing shortages, quantity discounts, inflation etc.

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